THE ALPHA

I was teaching the first semester of calculus in a large lecture hall and when I came to class one day there was piece of paper on the lectern on which the following was written:

Calculus is a monster. A Stephen Speilburg special effect. A voracious Gargantua, a meat eating Maha Kali, a bulldozer, downtown L.A. at rush hour, a colony of worms at the edge of a swamp, Twisted Sister's latest album played backwards, Isaac Newton's epilepsy, blindman's bluff with the Marquis de Sade, a Harvey Wallbanger and kielbasa breakfast, the secret language of lung fish, AND a mother of twins, a hole in the wall, lipstick graffiti on the men's room mirror, a nickel on the sidewalk, a nanosecond beneath an apple, apple sauce, the acceleration due to gravity, the gravity due to acceleration, a firefly in moonless midnight, the fluttering of bat wings, loft, the exhalation of the earth, the swelling of clouds, the chords of bassoons, the rippling of a web when the spider plucks, Beethoven's toy, sweet Venusian penuche, an afternoon in an English garden, tea stains on the lily.

I asked the class if they knew where this came from but they claimed no knowledge and I do not, to this day, know who the author was.

THE BEGINNING

There is some reason why you opened this book and are now reading these words. I am going to assume that the reason is the word ‘Calculus’ in the title, realizing that some of you may have picked it up randomly and have no idea what the title even is. No matter why you are here, I want to tell you a little bit about this book.

I am a teacher. Good, bad, or indifferent, that is what I am. While indulging my basic nature during thirty some odd years of teaching, I developed an opinion about mathematics in general and calculus in particular. This book is my opinion of the two basic concepts of calculus, the derivative and the definite integral, and the mathematics that surrounds them.

It seems to me that there are two major decisions to be made when writing a book on calculus: which topics and what kind of presentation.

The author must decide which topics to include and there are so many possible topics in calculus that the limitation to doing them all becomes the physical strength of the reader to hold the book. I have decided to present as few topics as possible, the definition of the derivative and the definition of the definite integral. I have to introduce some other things along the way, such as functions and lines, but the basic derivative and integral are the only two calculus topics. I do not talk about the rules of differentiation and the few derivatives that I do compute are done almost an aside. There are no problem sets. I use very few functions and many of those that I do introduce, I do so out of a personal indulgence to include some of my dear friends, and not because they are necessary to the book.

The author must decide how to present the topics. This is the tough one. Calculus can be presented as an abstract mathematical theory, and believing deeply in freedom of expression, I would certainly defend anyone’s right to do so. On the other hand, I think of the people who came up with calculus, Sir Isaac Newton in particular, as natural scientists and not pure mathematicians. I think of calculus as being developed to describe and solve problems in physics and consequently I think physics is its natural context. If the physical world is taken out of calculus, then, in my opinion, the baby has been thrown out, and the bath water kept. I develop the two calculus topics I have chosen in the context of searching for a mathematical description of a falling rock.

Any consideration of presentation must include what to do about proofs. It is not possible to prove everything but it seems as though a book on mathematics should prove something. I decided to include the proofs I like and omit the proofs I don’t like.

In the world at large, a proof is acceptable if it is found acceptable by the mathematical community and so the definition of a proof is side stepped. Mathematicians know a good proof when they see one. My proofs are proofs only under the broadest interpretation of the word.

One might say that my proofs use all the methods I was told never to use.

I base proofs on physical intuition. For example, I say that if the speed of a car is always greater than 60 miles/hour then its average speed is greater than 60 miles/hour. I believe it.

I base proofs on examples. If I have what seems to be a general example that satisfies the hypothesis of a theorem and the theorem holds for this example, I say that the example proves the theorem. If someone walks up to me on the street and asks me to show them why some mathematical statement is true, I give them an example of its truth.

I base proofs on pictures. If I have been told once, I have been told a thousand times not to use pictures in proofs. I have been shown examples of how pictures can lead to a false proof. I do it anyway. If someone walks up to me on the street and asks me to show them why some mathematical statement is true, I will demonstrate the truth with a picture.

I say that if something is true for the first few natural numbers, then it is true for all natural numbers. This may be my most egregious method of proof but it is also very natural for our species. Before the solar system was understood, people expected the sun to come up each morning even though they had only observed the event a finite number of times.

One of the deductions that I made as a child was that while forbidden activities had an element of danger, they were also fun. I am not doing rigorous mathematics, I am living dangerously and having fun.

My idea of proof is an argument that engenders belief. My proofs are what goes through my mind when I am convincing myself some result is true. When I am confronted with a mathematical proposition, I first ask myself, "Why is this true? What makes it work?" My proofs are answers to these questions and I hope that they will contain the germ of a formal proof, whatever that is.

The explanation part of a calculus book can range from no explanation at all to starting the explanation with the axioms of set theory and the rules of logic. This book is my stab at explanation. It expresses the way I think about mathematics in general and calculus in particular. It works for me and I am arrogant enough to think that it might work for someone else.

Calculus is packed full of techniques and I have dealt most of these techniques the same fate as proofs I don't like, although for a different reason. I love the techniques and their attendant problems. I do not talk about technique because my interest is in the ideas behind the techniques and I don't want to hide the ideas behind a forest of technique. Of course, I include some mathematical technique because the mathematical expression of the ideas is both the beauty and the point of calculus. As a final comment, I would suggest that this book be read and not studied.